Binomial Expansion (1.11.C) + Polar AROC (3.15) Homework

1. Which one of the following is equivalent to $(3x-2)^5$?
   (A) $(3x)^5-(2)^5$

   (B) $(3x)^5-(3x)^4(2)+(3x)^3(2)^2-(3x)^2(2)^3+(3x)(2)^4-(2)^5$

   (C) $(3x)^5-5(3x)^4(2)+10(3x)^3(2)^2-10(3x)^2(2)^3+5(3x)(2)^4-(2)^5$

   (D) $(3x)^5-5(3x)^4(2)+20(3x)^3(2)^2-30(3x)^2(2)^3+60(3x)(2)^4-(2)^5$

   (E) $(3x)^5-4(3x)^4(2)+6(3x)^3(2)^2-6(3x)^2(2)^3+4(3x)(2)^4-(2)^5$

2. The binomial theorem may be used to expand the polynomial function p, given by $p(x)=(2x+1)^6$. What is the coefficient of the x^2 term in the expanded polynomial?

3. The binomial theorem may be used to expand the polynomial function p, given by $p(x)=(x-3)^4$. What is the coefficient of the x^2 term in the expanded polynomial?

4. The binomial theorem may be used to expand the polynomial function p, given by $p(x)=(x+4)^5$. What is the coefficient of the x^3 term in the expanded polynomial?

5. The binomial theorem may be used to expand the polynomial function p, given by $p(x)=(2x-3)^4$. What is the coefficient of the x term in the expanded polynomial?

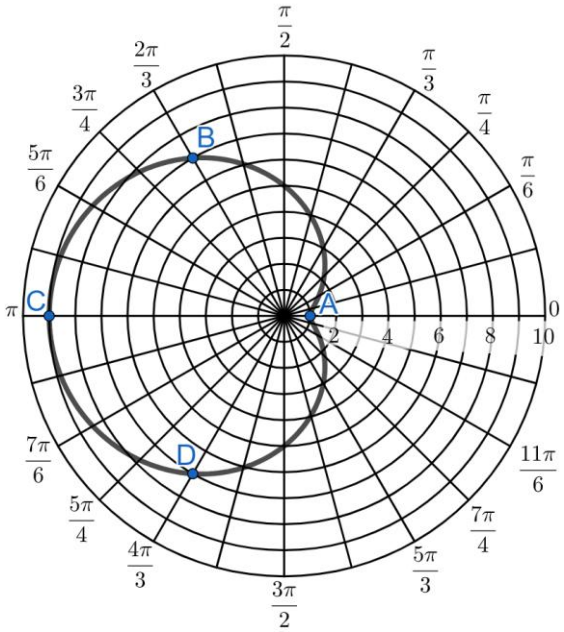
6. The binomial theorem may be used to expand $(3x+2y)^5$. What is the coefficient of the xy^4 term in the expansion?

7. The binomial theorem can be used to expand the polynomial function p, given by $p(x) = (2x - 1)^5$. Expand this expression to write $p(x)$ in standard form.

8. The binomial theorem can be used to expand the polynomial function p, given by $p(x) = (x^3 - 2)^5$. Expand this expression to write $p(x)$ in standard form.

9. The function f is given by $f(\theta) = 4 + 2\sin(3\theta)$.
Find the average rate of change of $f(\theta)$ with respect to θ over the interval $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$.

10. In the polar coordinate system, the graph of a polar function $r = f(\theta)$ is shown to the right with a domain of all real values of θ for $0 \leq \theta \leq 2\pi$. On this interval of θ, the graph has no holes, passes through each point exactly once, and as θ increases, the graph passes through the labelled points A, B, C, and D, in that order. On which of the following intervals is the average rate of change of r with respect to θ least?
(A) From A to B
(B) From A to C
(C) From B to D
(D) From C to D



11. The function f is given by $f(\theta) = 2 + 2\cos(\theta)$. Find the average rate of change of $f(\theta)$ with respect to θ over the interval $0 \leq \theta \leq \frac{\pi}{2}$. Use this average rate of change to estimate $f\left(\frac{\pi}{4}\right)$.

Binomial Expansion and Polar AROC HW Answers

1. C
2. 60
3. 54
4. 160
5. -216
6. 240
7. $32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1$
8. $x^{15} - 10x^{12} + 40x^9 - 80x^6 + 80x^3 - 32$
9. $\frac{f\left(\frac{\pi}{3}\right)-f\left(\frac{\pi}{6}\right)}{\frac{\pi}{3}-\frac{\pi}{6}} = \frac{4-6}{\frac{\pi}{6}} = -\frac{12}{\pi}$
10. D
11. Average Rate of Change: $\frac{f\left(\frac{\pi}{2}\right)-f(0)}{\frac{\pi}{2}-0} = \frac{2-4}{\frac{\pi}{2}} = -\frac{4}{\pi}$

    Estimation:

$$\frac{f\left(\frac{\pi}{4}\right) - f(0)}{\frac{\pi}{4} - 0} \approx -\frac{4}{\pi}$$
$$\frac{f\left(\frac{\pi}{4}\right) - 4}{\frac{\pi}{4}} \approx -\frac{4}{\pi}$$
$$f\left(\frac{\pi}{4}\right) - 4 \approx -\frac{4}{\pi} \cdot \frac{\pi}{4}$$
$$f\left(\frac{\pi}{4}\right) - 4 \approx -1$$
$$f\left(\frac{\pi}{4}\right) \approx 3$$

    Alternatively, use the line that passes through $(0,4)$ with a slope of $-\frac{4}{\pi}$

$$y = -\frac{4}{\pi}x + 4$$
$$f(\theta) \approx -\frac{4}{\pi}\theta + 4$$
$$f\left(\frac{\pi}{4}\right) \approx -\frac{4}{\pi}\left(\frac{\pi}{4}\right) + 4 = 3$$